Commutative and anti-commutative matrices

Two matrices A and B are said to commutative if AB=BA.

Two matrices A and B are said to anti-commutative if AB+BA=0.

Example for commutative

$$A.B = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}.\begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 5\times 2+1\times 4 & 5\times 4+1\times 2 \\ 1\times 2+5\times 4 & 1\times 4+5\times 2 \end{pmatrix} = \begin{pmatrix} 14 & 22 \\ 22 & 14 \end{pmatrix}$$

$$\&\ \ B.A = \begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}.\begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 2\times 5+4\times 1 & 4\times 5+2\times 1 \\ 2\times 1+4\times 5 & 4\times 1+2\times 5 \end{pmatrix} = \begin{pmatrix} 14 & 22 \\ 22 & 14 \end{pmatrix}$$

Since AB=BA. So A and B are said to commutative

Example for anti-commutative.

$$A.B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0\times 1+(-1)\times 0 & 0\times 0+(-1)\times 1 \\ 1\times 1+0\times 0 & 1\times 0+0\times(-1) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

But $\quad B.A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1\times 0+0\times(-1) & 0\times 0+(1)\times(-1) \\ 1\times 1+0\times 0 & 0\times 1+(-1)\times 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

then $A.B + B.A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0+0 & 1-1 \\ 1-1 & 0=0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

So those two matrices anti-commute.

**** Show that , $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$are anti-commute.

******The following two matrices are commute

i. $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 0 \\ -1 & -1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -6 \\ 3 & 2 & 9 \\ -1 & -1 & -4 \end{bmatrix}$ ans: $AB = BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ii. $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ -1 & 2 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} \frac{2}{3} & 0 & -\frac{1}{3} \\ -\frac{3}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{7}{15} & -\frac{1}{5} & \frac{1}{15} \end{bmatrix}$ ans: $AB = BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$